C2 Exercise 1E (remainder theorem)

1 a Find the remainder when $4x^3 - 5x^2 + 7x + 1$ is divided by $(x-2)$

$$f(2) = 4(2)^3 - 5(2)^2 + 7(2) + 1$$

$$= 32 - 20 + 14 + 1$$

$$= 27$$

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1b Find the remainder when $2x^5 - 32x^3 + x - 10$ is divided by $(x-4)$

$$f(4) = 2(4)^5 - 32(4)^3 + (4) - 10$$

$$= 2(1024) - 32(64) + 4 - 10$$

$$= 2048 - 2048 - 6 = -6$$

The remainder is -6

C2 Ex1E

1c Find the remainder when $-2x^3+6x^2+5x-3$ is divided by $(x+1)$

$$f(-1) = -2(-1)^3 + 6(-1)^2 + 5(-1) - 3$$

$$= 2 + 6 - 5 - 3 = 0$$

The remainder is 0, so $x+1$ is a factor of $f(x)$

C2 Ex 1E

1d Find the remainder when $7x^3+6x^2-45x+1$ is divided by $(x+3)$

$f(-3) = 7(-3)^3 + 6(-3)^2 - 45(-3) + 1$

$= 7(-27) + 6(9) + 135 + 1$

$= -189 + 54 + 1 + 135$

$= 190 - 189$

$= 1$

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1e Find the remainder when $4x^4 - 4x^2 + 8x - 1$ is divided by $(2x - 1)$

$2x - 1 = 0 \Rightarrow x = 1/2$ so find $f(\frac{1}{2})$.

$$f(\tfrac{1}{2}) = 4\left(\tfrac{1}{2}\right)^4 - 4\left(\tfrac{1}{2}\right)^2 + 8\left(\tfrac{1}{2}\right) - 1$$

$$= \frac{4}{16} - \frac{4}{4} + 4 - 1$$

$$= \frac{1}{4} + 2$$

$$= 2\tfrac{1}{4}$$

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1e Find the remainder when $4x^4 - 4x^2 + 8x - 1$ is divided by $(2x - 1)$

This time by division:

$$\begin{array}{r} 2x^3 + x^2 - \tfrac{3}{2}x + \tfrac{13}{4} \\ 2x-1 \overline{) 4x^4 + 0x^3 - 4x^2 + 8x - 1} \\ -(4x^4 - 2x^3) \\ \hline 2x^3 - 4x^2 \\ -(2x^3 - x^2) \\ \hline -3x^2 + 8x \\ -(-3x^2 + \tfrac{3}{2}x) \\ \hline \tfrac{13}{2}x - 1 \\ -(\tfrac{13}{2}x - \tfrac{13}{4}) \\ \hline -1 + \tfrac{13}{4} \end{array}$$

but $-1 + \tfrac{13}{4} = \frac{-4+13}{4}$

$= \frac{9}{4}$

$= 2\tfrac{1}{4}$

as required.

C2 Ex 1E

1f Find the remainder when $243x^4 - 27x^3 - 3x + 7$ is divided by $(3x-1)$

$$3x - 1 = 0 \Rightarrow x = 1/3$$

$$243x^6 - 27x^3 - 3x + 7 = 3(3x)^4 - (3x)^3 - (3x) + 7$$

This is my cunning plan to avoid using fractions. If you didn't spot it (and I expect you didn't) you can still evaluate this using fractions / a calculator.

$$3\left(\tfrac{1}{3}\right) = 1 \Rightarrow f\left(\tfrac{1}{3}\right) = 3(1)^4 - (1)^3 - (1) + 7$$

$$= 8$$

C2 Ex 1E

1g Find the remainder when $64x^3+32x^2-16x+9$ is divided by $(4x+1)$

$4x+1=0$ when $x=-\frac{1}{4}$ so evaluate $f(-\frac{1}{4})$

$$64x^3+32x^2-16x+9 = (4x)^3+2(4x)^2-4(4x)+9$$

Did you see that coming this time?

$$\text{so } f(-\tfrac{1}{4}) = (-1)^3+2(-1)^2-4(-1)+9$$

$$= -1+2+4+9$$

$$= 14$$

C2 Ex 1E

1h Find the remainder when $81x^3 - 81x^2 + 9x + 6$ is divided by $(3x-2)$

$3x - 2 = 0$ when $x = 2/3$ so find $f(2/3)$

Now $f(x) = 81x^3 - 81x^2 + 9x + 6 = 3(3x)^3 - 9(3x)^2 + 3(3x) + 6$

So $f(2/3) = 3(2)^3 - 9(2)^2 + 3(2) + 6$

$= 24 - 36 + 6 + 6$

$= 0.$

So $(3x-2)$ is a factor of $81x^3 - 81x^2 + 9x + 6$

C2 Ex 1E

1 i Find the remainder when $243x^6 - 780x^2 + 6$ is divided by $(3x+4)$

$3x + 4 = 0 \Rightarrow x = -4/3$ so need $f\left(\frac{-4}{3}\right)$

tried using the $243x^6 = \frac{(3x)^6}{3}$ trick, but it doesn't help much here.

$$f\left(\frac{-4}{3}\right) = 243\left(\frac{-4}{3}\right)^6 - 780\left(\frac{-4}{3}\right)^2 + 6$$

$$= \frac{\cancel{243}^{1} \times 4096}{\cancel{729}\ 3} - \frac{780 \times 16}{9} + \frac{54}{9}$$

$$\begin{array}{r} 780 \\ \times\ 16 \\ \hline 12480 \\ \hline 54 \end{array}$$

$$= \frac{4096}{3} - \frac{12426}{9} = \frac{4096}{3} - \frac{4142}{3}$$

$$= -\frac{46}{3} = -15\tfrac{1}{3}$$

C2 Ex 1E

1j Find the remainder when $125x^4 + 5x^3 - 9x$ is divided by $(5x+3)$

$5x + 3 = 0 \Rightarrow x = -3/5$ so find $f(-3/5)$

$$f(-3/5) = 125\left(\frac{-3}{5}\right)^4 + 5\left(\frac{-3}{5}\right)^3 - 9\left(-\frac{3}{5}\right)$$

$$= \frac{125 \times 81}{625} + \frac{5 \times -27}{125} + \frac{9 \times 3}{5}$$

$$= \frac{81}{5} + \frac{-27}{25} + \frac{27}{5}$$

$$= \frac{405 - 27 + 135}{25} = \frac{513}{25} = \frac{2052}{100}$$

$$= 20.52$$

C2 Ex 1E

2 When $2x^3 - 3x^2 - 2x + a$ is divided by $(x-1)$ the remainder is -4. Find the value of a.

$f(1) = -4$

$\Rightarrow \quad 2(1)^3 - 3(1)^2 - 2(1) + a = -4$

$\Rightarrow \quad 2 - 3 - 2 + a = -4$

$\Rightarrow \quad a = -1$

C2 Ex 1E

3 When $-3x^3 + 4x^2 + bx + 6$ is divided by $(x+2)$ the remainder is 10. Find b.

$$f(-2) = 10 \quad \Rightarrow \quad -3(-2)^3 + 4(-2)^2 + b(-2) + 6 = 10$$

$$\Rightarrow \quad 24 + 16 - 2b + 6 = 10$$

$$\Rightarrow \quad 2b = 46 - 10 = 36$$

$$\Rightarrow \quad b = 18$$

C2 Ex 1E

4 When $16x^3 - 32x^2 + cx - 8$ is divided by $(2x-1)$ the remainder is 1. Find c.

$2x - 1 = 0 \Rightarrow x = 1/2$ so equate $f(\tfrac{1}{2}) = 1$

$$16\left(\tfrac{1}{2}\right)^3 - 32\left(\tfrac{1}{2}\right)^2 + c\left(\tfrac{1}{2}\right) - 8 = 1$$

$$2 - 8 + \tfrac{1}{2}c - 8 = 1$$

$$\tfrac{1}{2}c = 15$$

$$c = 30$$

C2 Ex 1E

5 Show that $(x-3)$ is a factor of $x^6 - 36x^3 + 243$

$x-3$ is a factor of $f(x) \Leftrightarrow f(3) = 0$

$$(3)^6 - 36(3)^3 + 243 = 729 - 972 + 243$$

$$= 972 - 972$$

$$= 0$$

Since $f(3) = 0$ $(x-3)$ is a factor of $f(x)$, by the factor theorem

C2 Ex 1E

6 Show that $(2x-1)$ is a factor of $2x^3+17x^2+31x-20$

$2x-1=0$ when $x=\frac{1}{2}$ so find $f(\frac{1}{2})$

$$f(\tfrac{1}{2}) = 2\left(\tfrac{1}{2}\right)^3 + 17\left(\tfrac{1}{2}\right)^2 + 31\left(\tfrac{1}{2}\right) - 20$$

$$= \frac{2}{8} + \frac{17}{4} + \frac{31}{2} - 20$$

$$= \frac{18}{4} + \frac{62}{4} - \frac{80}{4}$$

$$= \frac{80-80}{4} = 0$$

Since $f(\frac{1}{2})=0$ $(2x-1)$ is a factor of $f(x)$

C2 Ex 1E

7 $f(x) = x^2 + 3x + q$. Given $f(2) = 3$ find $f(-2)$

(i) find q. $f(2) = (2)^2 + 3(2) + q = 3$

$$4 + 6 + q = 3$$

$$q = -7$$

(ii) $f(-2) = (-2)^2 + 3(-2) - 7$

$= 4 - 6 - 7$

$= -9$

C2 Ex 1E

8 $g(x) = x^3 + ax^2 + 3x + 6$. Given $g(-1) = 2$, find the remainder when $g(x)$ is divided by $(3x-2)$

$$g(-1) = (-1)^3 + a(-1)^2 + 3(-1) + 6 = 2$$

$$\Rightarrow -1 + a - 3 + 6 = 2$$

$$\Rightarrow a = 0$$

remainder when $g(x)$ is divided by $(3x-2)$ is $g(2/3)$

$$g(2/3) = \left(\frac{2}{3}\right)^3 + 0 + 3\left(\frac{2}{3}\right) + 6$$

$$= \frac{8}{27} + 2 + 6 = 8\,8/27$$

C2 Ex 1E

9 The expression $2x^3 - x^2 + ax + b$ gives remainder 14 when divided by $(x-2)$ and a remainder of -86 when divided by $(x+3)$. Find the value of a and b.

$f(2) = 14$ so $2(2)^3 - (2)^2 + 2a + b = 14$

$\Rightarrow 16 - 4 + 2a + b = 14$

$\Rightarrow 2a + b = 2$ ①

$f(-3) = -86$ so $2(-3)^3 - (-3)^2 - 3a + b = -86$

$\Rightarrow -54 - 9 - 3a + b = -86$

$\Rightarrow -3a + b = -23$ ②

① − ② $\Rightarrow$ $5a = 25$ $\Rightarrow$ $a = 5$

① $\Rightarrow$ $2(5) + b = 2$ $\Rightarrow$ $b = -8$

C2 Ex1E

10 The expression $3x^3 + 2x^2 - px + q$ is divisible by $(x-1)$ but leaves a remainder of 10 when divided by $(x+1)$. Find the value of a and b. ! They mean p and q. at least I hope they do.

$f(1) = 0 \Rightarrow 3 + 2 - p + q = 0$

$q - p = -5$ ①

$f(-1) = 10 \Rightarrow -3 + 2 + p + q = 10$

$p + q = 11$ ②

① + ② $\Rightarrow 2q = 6 \Rightarrow q = 3$

② $\Rightarrow p + 3 = 11 \Rightarrow p = 8$

$p = 8,\ q = 3.$